APPLICATION OF THE STRAIN ENERGY DAMAGE DETECTION METHOD TO PLATE-LIKE STRUCTURES

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ABSTRACT

In this paper the problem of using measured modal parameters to detect and locate damage in plate-like structures is investigated. Many methods exist for locating damage in a structure given the modal properties before and after damage. Unfortunately, many of these methods require a correlated finite element model or mass normalized mode shapes. If the modal properties are obtained using ambient excitation then the mode shapes will not be mass normalized. In this paper a method based on the changes in the strain energy of the structure will be discussed. This method has been successfully applied to beam-like structures, that is, structures characterized by one-dimensional curvature. In this paper the method will be generalized to plate-like structures that are characterized by two-dimensional curvature. This method only requires the mode shapes of the structure before and after damage. To evaluate the effectiveness of the method it will be applied to simulated data.

NOMENCLATURE

D	$= Eh^3/12(1-\nu^2)$ = bending stiffness of a plate
EI	Flexural rigidity
F_{ij}	Fractional energy associated with sub-region j due to the i^{th} mode
F_{ijk}	Fractional energy associated with sub-region jk due to the i^{th} mode
N_d	Number of divisions in beam-like structure
N_x	Number of divisions in the x-direction of a plate-like structure
N_y	Number of divisions in the y-direction of a plate-like structure
U	Strain energy
U_i	Strain energy associated with the i^{th} mode
U_{ij}	Strain energy associated with sub-region j due to the i^{th} mode
U_{ijk}	Strain energy associated with sub-region jk due to the i^{th} mode
x, y, z	Translational coordinates
u, v, w	Displacements in x, y and z directions
β_k	Damage index for sub-region k
β_{jk}	Damage index for sub-region jk
Z_k	Normalized damage index for sub-region k
$\bar{\beta}_k$, σ_k	mean and standard deviation of β_k
ψ_i	i^{th} mode shape
$( )^*$	Indicates a quantity calculated using the damaged mode shapes, ψ_i^*

INTRODUCTION

Significant work has been done in the area of detecting damage in structures using changes in the dynamic response of the structure. Since the natural frequencies and mode shapes of a structure are dependent on the mass and stiffness distributions any subsequent changes in them should, theoretically, be reflected in changes in the frequency and mode shapes of the structure. The problem of using measured frequencies and mode shapes and their sensitivity to damage is a question not to be addressed in this paper. An extensive literature review [1] of the state of the art of damage detection and health monitoring from vibration characteristics has recently been published. From this review it is clear that there are a large number of proposed methods of detecting damage from vibration characteristics but, unfortunately, many of these methods require a correlated finite element model and/or mass normalized mode shapes. If the modal properties are obtained using ambient excitation, as would most likely be the case for a remote, automated health monitoring system, then the mode shapes will not be mass normalized. The method proposed in this paper avoids both of these problems.

In this paper an extension of a method proposed by Stubbs and Kim [2] will be presented. This method requires that the mode shapes before and after damage be known, but the modes do not need to be mass normalized and only a few modes are required. The original formulation by Stubbs and Kim was primarily for beam-like structures that are characterized by one-dimensional curvature. In this paper the method will be generalized to plate-like structures that are characterized by two-dimensional curvature. To examine limitations of the method it will be applied to several sets of simulated data and comparisons will be made between applying the original formulation to a series of slices of the structure verses the true two-dimensional formulation.

THEORY

For completeness the derivation of the damage indicator will be shown for both beam-like and plate-like structures.

Beam-like structures

The strain energy of a Bernoulli-Euler beam is given by

$$U=\frac{1}{2}\int_0^{\ell} EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{1}$$

For a particular mode shape, $\psi_i(x)$, the energy associated with that mode shape is

$$U_i=\frac{1}{2}\int_0^{\ell} EI\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx \tag{2}$$

If the beam is subdivided into N_d divisions as shown in Figure 1, then the energy associated with each sub-region j due to the ith mode is given by

$$U_{ij}=\frac{1}{2}\int_{a_j}^{a_{j+1}} (EI)_j\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx \tag{3}$$

The fractional energy is therefore

$$F_{ij}=\frac{U_{ij}}{U_i} \tag{4}$$

and

$$\sum_{j=1}^{N_d} F_{ij}=1 \tag{5}$$

Similar quantities can be defined for a damaged structure and are given by Eq. (6-9)

$$U_i^*=\frac{1}{2}\int_0^{\ell} EI^*\left(\frac{\partial^2 \psi_i^*}{\partial x^2}\right)^2 dx \tag{6}$$

$$U_{ij}^*=\frac{1}{2}\int_{a_j}^{a_{j+1}} (EI)_j^*\left(\frac{\partial^2 \psi_i^*}{\partial x^2}\right)^2 dx \tag{7}$$

$$F_{ij}^*=\frac{U_{ij}^*}{U_i^*} \tag{8}$$

and

$$\sum_{j=1}^{N_d} F_{ij}=\sum_{j=1}^{N_d} F_{ij}^*=1 \tag{9}$$

By choosing the sub-regions to be relatively small, the flexural rigidity for the jth sub-region, EI_j is roughly constant and F_{ij}^* becomes

$$F_{ij}^*=\frac{(EI)_j^*\int_{a_j}^{a_{j+1}}\left(\frac{\partial^2 \psi_i^*}{\partial x^2}\right)^2 dx}{U_i^*} \tag{10}$$

If we assume that the damage is primarily located at a single sub-region then the fractional energy will remain relatively constant in undamaged sub-regions and $F_{ij}^*=F_{ij}$. For a single damaged location at sub-region j=k we find

$$\frac{(EI)_k\int_{a_k}^{a_{k+1}}\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx}{U_i}=\frac{(EI)_k^*\int_{a_k}^{a_{k+1}}\left(\frac{\partial^2 \psi_i^*}{\partial x^2}\right)^2 dx}{U_i^*} \tag{11}$$

If we assume that EI is essentially constant over the length of the beam for both the undamaged and damaged modes Eq. (11) can be rearranged to give an indication of the change in the flexural rigidity of the sub-region as shown in Eq. (12)

$$\frac{(EI)_k}{(EI)_k^*}=\frac{\int_{a_k}^{a_{k+1}}\left(\frac{\partial^2 \psi_i^*}{\partial x^2}\right)^2 dx \Big/ \int_0^{\ell}\left(\frac{\partial^2 \psi_i^*}{\partial x^2}\right)^2 dx}{\int_{a_k}^{a_{k+1}}\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx \Big/ \int_0^{\ell}\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx}\equiv\frac{f_{ik}^*}{f_{ik}} \tag{12}$$

In order to use all the measured modes, m, in the calculation, the damage index for sub-region k is defined to be

$$\beta_k=\frac{\sum_{i=1}^{m} f_{ik}^*}{\sum_{i=1}^{m} f_{ik}} \tag{13}$$

One advantage to the formulation shown in Eqs. 12 and 13 is that the modes do not need be normalized. Assuming that the collection of the damage indices, β_k, represent a sample population of a normally distributed random variable, a normalized damage index is obtained using Eq. (14)

$$Z_k = \frac{\beta_k - \bar{\beta}_k}{\sigma_k} \tag{14}$$

where $\bar{\beta}_k$ and σ_k represent the mean and standard deviation of the damage indices, respectively. In this paper it will be assumed that normalized damage indices with values greater than two are associated with potential damage locations.

Plate-like structures

The strain energy of a plate is given by Eq. 15 [3].

$$U = \frac{D}{2}\int_0^b\int_0^a \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 dxdy \tag{15}$$

For a particular mode shape, $\psi_i(x,y)$, the energy associated with that mode shape is

$$U_i = \frac{D}{2}\int_0^b\int_0^a \left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \psi_i}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)\left(\frac{\partial^2 \psi_i}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 \psi_i}{\partial x \partial y}\right)^2 dxdy \tag{16}$$

If the plate is subdivided into N_x subdivisions in the x direction and N_y subdivisions in the y direction as shown in Figure 2 then the energy associated with sub-region jk for the i[th] mode is given by

$$U_{ijk} = \frac{D_{jk}}{2}\int_{b_k}^{b_{k+1}}\int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \psi_i}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)\left(\frac{\partial^2 \psi_i}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 \psi_i}{\partial x \partial y}\right)^2 dxdy \tag{17}$$

so

$$U_i = \sum_{k=1}^{N_y}\sum_{j=1}^{N_x} U_{ijk} \tag{18}$$

and the fractional energy at location jk is defined to be

$$F_{ijk} = \frac{U_{ijk}}{U_i} \tag{19}$$

and

$$\sum_{k=1}^{Ny}\sum_{j=1}^{Nx} F_{ijk} = 1 \tag{20}$$

Similar expressions can be written using the modes of the damaged structure, ψ_i^*. Using arguments similar to the ones used for beam-like structures a ratio of parameters can be determined that is indicative of the change of stiffness in the structure as shown in Eq. 21-22.

$$\frac{D_{jk}}{D_{jk}^*} = \frac{f_{ijk}^*}{f_{ijk}} \tag{21}$$

where

$$f_{ijk} = \frac{\int_{b_k}^{b_{k+1}}\int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \psi_i}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)\left(\frac{\partial^2 \psi_i}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 \psi_i}{\partial x \partial y}\right)^2 dxdy}{\int_0^b\int_0^a \left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \psi_i}{\partial y^2}\right)^2 + 2\nu\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)\left(\frac{\partial^2 \psi_i}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 \psi_i}{\partial x \partial y}\right)^2 dxdy} \tag{22}$$

and an analogous term f_{ijk}^* can be defined using the damaged mode shapes. In order to account for all measured modes, the following formulation for the damage index for sub-region jk is used

$$\beta_{jk} = \frac{\sum_{i=1}^{m} f_{ijk}^*}{\sum_{i=1}^{m} f_{ijk}} \tag{23}$$

Once again a normalized damage index can be found using Eq. (14).

RESULTS

Both algorithms discussed in the theory section can be applied to detect damage in plate-like structures. The algorithm derived assuming plate-like behavior (two-dimensional curvature) can obviously be applied directly. To use the algorithm formulated assuming one-dimensional curvature the structure must be divided into slices and the algorithm needs to be applied to each slice individually. The normalized damage index is then determined using the average and standard deviation of all the damage indices from all the slices. The advantage of this approach is that it is computational more efficient than the 2-D algorithm. Regardless of the method chosen several additional parameters must be chosen including the number of modes and the number of subdivisions to be used.

Several sets of simulated data were used to investigate the effectiveness of both approaches in locating damage in plate-like structures as well as to study the effect of changing the number of modes and subdivisions. The data was generated using a finite element model of a pinned-pinned plate with several elements reduced in stiffness to model damage. The finite element mesh is shown in Fig. 3. The plate was given pinned boundary conditions at y=-300 and y=300. The elements in the location of reduced stiffness are indicated in Fig. 3. The reduced stiffness was in the region 48<x<96, and 84<y<132 with the center of the damage being located at x=72, y=108. All the analysis was done using MATLAB.

The first case studied had the stiffness of four elements reduced by 25%. The results from this case are shown in Figs. 4-7. In all of these figures the normalized damage index is shown as a 3-D bar graph with values greater than 2 drawn in a darker color and are the likely locations of damage.

In Fig. 4 the damage index is shown by dividing the structure into slices in the longitudinal direction of the plate, that is, slices with constant x values, with 20 divisions per slice and using just one mode. It is clear from Fig. 4 that the algorithm does a fairly good job of locating the damage. The largest peak in Fig. 4 has its center at a location of (72,105). Increasing the number of modes to five did not noticeably change the results. In Fig. 5 the number of divisions per slice was increased to 40 and four modes are used. The results did not improve by increasing the divisions and the location of the damage was found to be roughly the same. When the structure was divided into slices in the transverse direction (i.e. constant y), 20 divisions/slice and using four modes, the damage was once again located as shown in Figure 6.

In Figure 7 the damage indices calculated using the algorithm derived for plate-like structures, one mode, and 20 divisions in each directions, are shown. Once again the damage is located fairly accurately. The two peaks in Fig. 7 are located at (64.8,105) and (79.2,105). Clearly the choice of the number of divisions will affect the location of the peak. Once again, increasing the number of divisions and modes does not significantly improve the results.

An example in which the method of dividing the structure into slices has several problems is examined in the second set of simulated data. In this case the stiffness was reduced by only 10%. The results of dividing the structure into longitudinal slices with 20 divisions/slice and using one and four modes are shown in Fig. 8 and 9 respectively. In this case the region of reduced stiffness was not located when using just one mode and when additional modes were included the damage indices were found to be large along the node line of the second natural mode. In Fig. 10-11 the results using the algorithm for plate-like structures is shown. In Fig. 10 only one mode was used and 20 divisions were used in both the x and y directions. From Fig. 10 it is clear that the area of reduced stiffness was not located. In Fig. 11 four modes and 20 divisions in each direction were used in the 2-D algorithm and clearly the general location of the damage has been identified.

In all of the examples used thus far it was assumed that the mode shapes were known exactly on a very fine grid of sensors. In actual practice this will obviously not be the case. A reduced set of data was used to determine how the results change using a coarser grid of sensors. In this case the stiffness was reduced 25% and the number of sensor locations was reduced from 338 to 56. The results from dividing the structure into longitudinal slices with 20 divisions/slice and using four modes is shown in Figure 12. Once again, when more than one mode is used, the algorithm incorrectly identified damage as being along a node line. Also, the resolution of this method is clearly limited by the number of slices that are available. Results from using the algorithm for 2-D curvature with four modes and 20 divisions in each direction are shown in Fig. 13 and the general area of the damage can be clearly seen.

One of the major difficulties associated with implementing the algorithms discussed in this paper was the calculation of the derivatives and integrals when the mode shape is known at a relatively small number of discrete locations. In both algorithms additional intermediate points were calculated by curve-fitting the data. The derivatives and integrals required by the algorithms were then calculated numerically.

CONCLUSIONS

A damage detection algorithm derived for structures whose modes are characterized by one-dimensional curvature has been generalized for plate-like structures that are characterized by two-dimensional curvature. The method only requires the mode shapes of the structure before and after damage and the modes do not need to be mass normalized making it very advantageous when using ambient excitation. The algorithm was found to be effective in locating areas with stiffness reductions as low as 10% using relatively few modes.

ACKNOWLEDGMENTS

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[2] Stubbs, N., J.-T. Kim, and C. R. Farrar, "Field Verification of a Nondestructive Damage Localization and Sensitivity Estimator Algorithm," Proceedings of the 13th International Modal Analysis Conference, pp. 210-218, 1995.

[3] Young, D., "Vibration of Rectangular Plates by the Ritz Method," *Journal of Applied Mechanics*, Dec. 1956, pp. 448-453

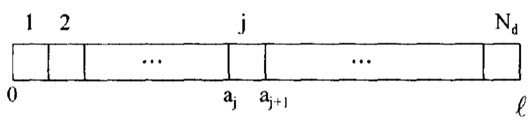

Figure 1 - A schematic illustrating a beam's N_d subdivisions.

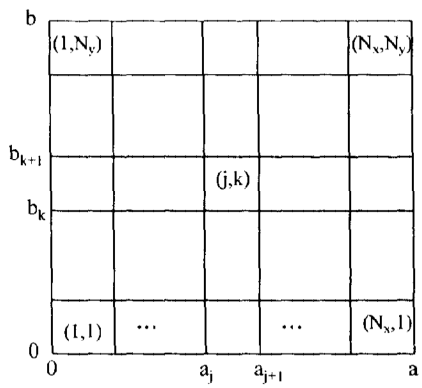

Figure 2 - A schematic illustrating a plate's N_x x N_y subregions.

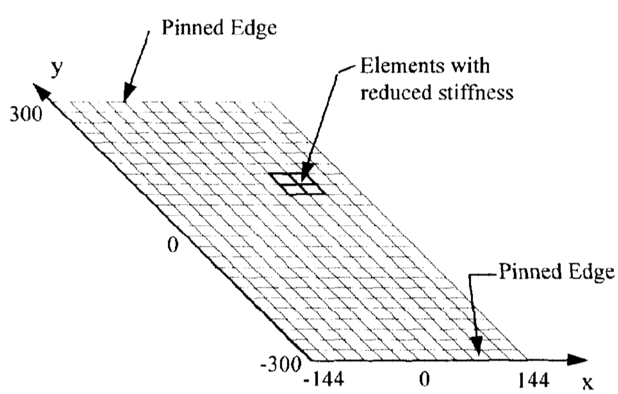

Figure 3 - Finite element mesh of a pinned-pinned plate with an area of reduced stiffness.

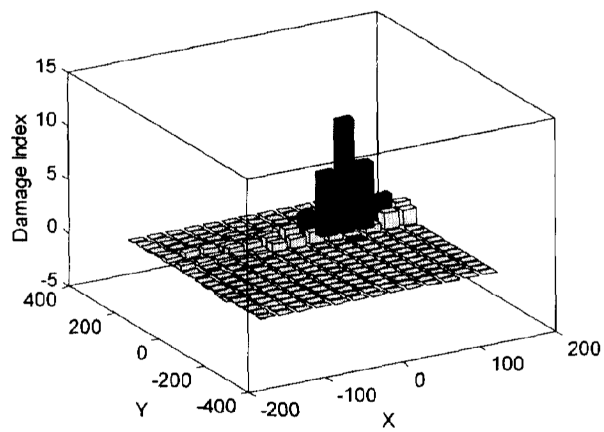

Figure 4 - Damage index for a plate with a region of 25% reduced stiffness. The plate was divided into longitudinal slices, 20 divisions per slice, and one mode was used in the algorithm.

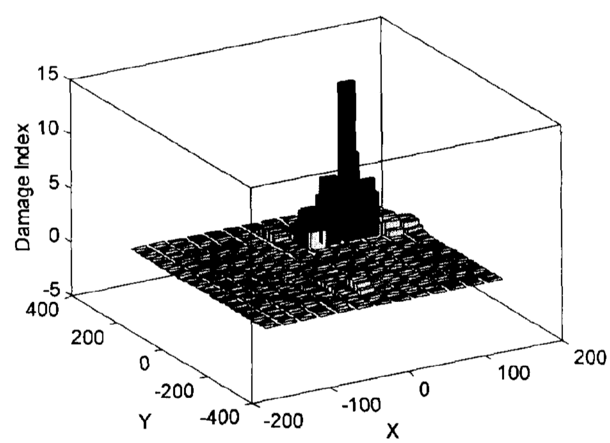

Figure 5 - Damage index for a plate with a region of 25% reduced stiffness. The plate was divided into longitudinal slices, 40 divisions per slice, and four modes were used in the algorithm.

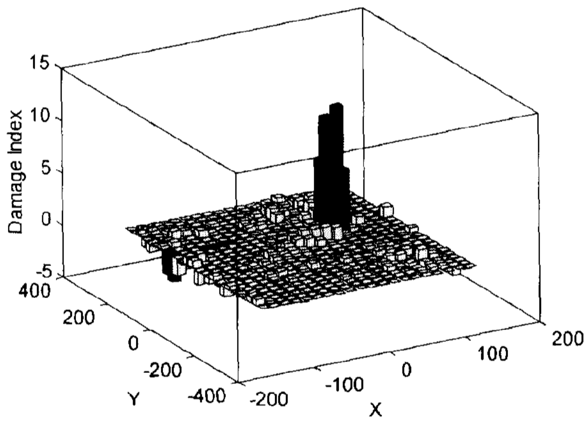

Figure 6 - Damage index for a plate with a region of 25% reduced stiffness. The plate was divided into transverse slices, 20 divisions per slice, and four modes were used in the algorithm.

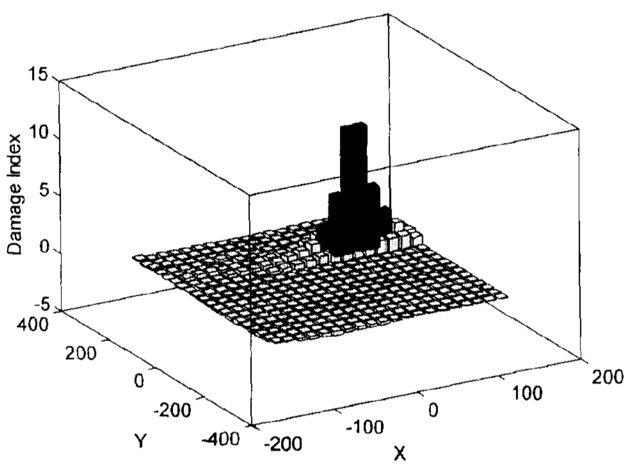

Figure 7 - Damage index for a plate with a region of 25% reduced stiffness. The plate was divided into 20 divisions in each direction and the 2-D algorithm was used with one mode.

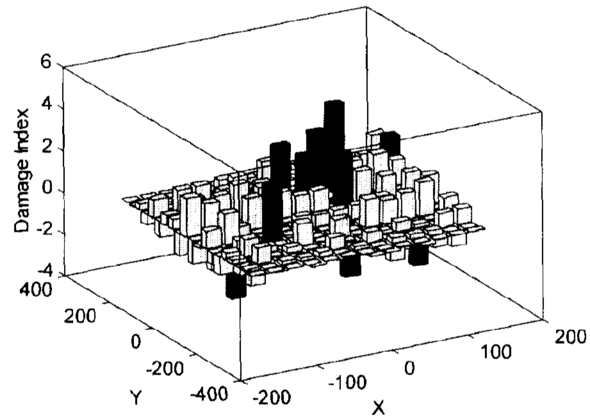

Figure 8 - Damage index for a plate with a region of 10% reduced stiffness. The plate was divided into longitudinal slices, 20 divisions per slice, and one mode was used in the algorithm.

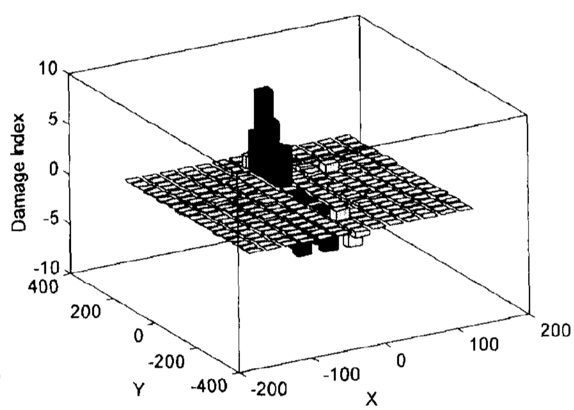

Figure 9 - Damage index for a plate with a region of 10% reduced stiffness. The plate was divided into longitudinal slices, 20 divisions per slice, and four modes were used in the algorithm.

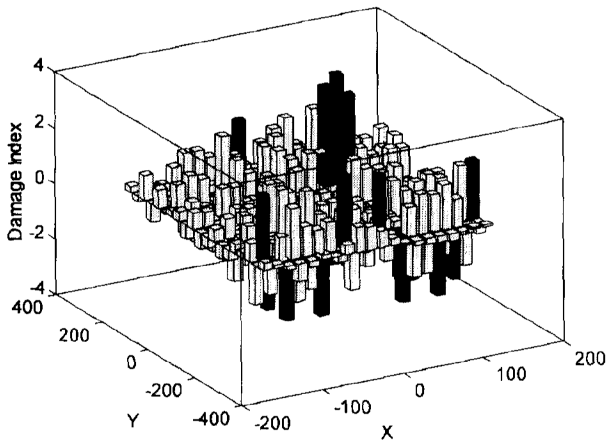

Figure 10 - Damage index for a plate with a region of 10% reduced stiffness. The plate was divided into 20 divisions in each direction and the 2-D algorithm was used with one mode.

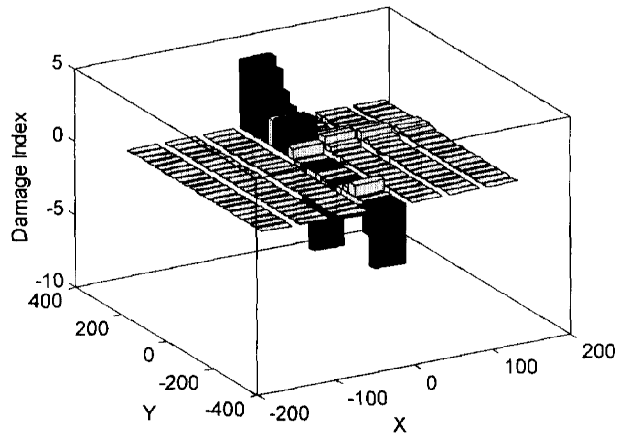

Figure 12 - Damage index for a plate with a region of 25% reduced stiffness using a reduced number of sensors. The plate was divided into longitudinal slices, 20 divisions per slice, and four modes were used in the algorithm.

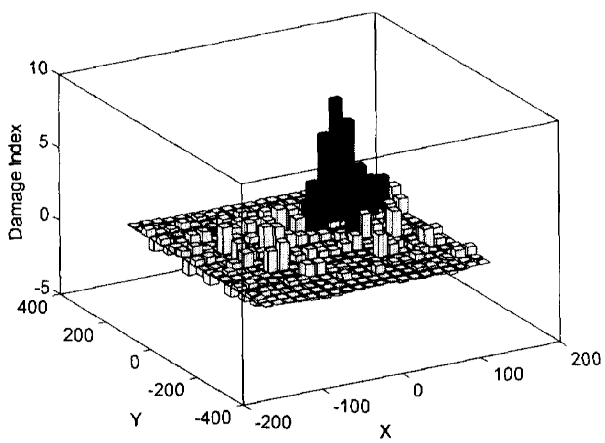

Figure 11 - Damage index for a plate with a region of 10% reduced stiffness. The plate was divided into 20 divisions in each direction and the 2-D algorithm was used with four modes.

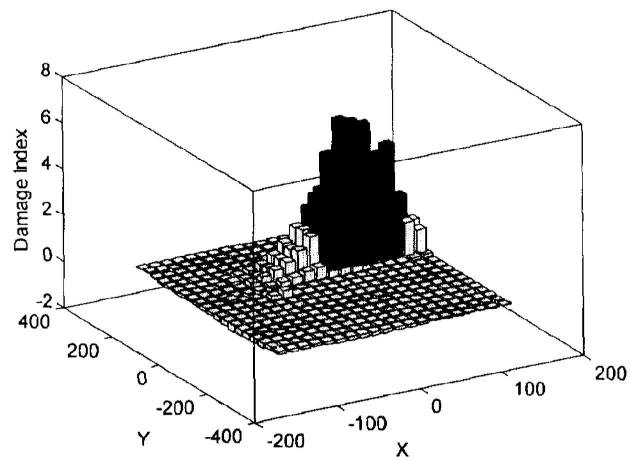

Figure 13 - Damage index for a plate with a region of 10% reduced stiffness. The plate was divided into 20 divisions in each direction and the 2-D algorithm was used with four modes.